Retirement Income University

[A MONTHLY COURSE ON RETIREMENT ECONOMICS **BY MOSHE A. MILEVSKY,** Ph.D.]

LESSON 3:

Sustainability and Ruin

As you are reading this in early April 2007, I am facing the daunting prospect of celebrating my 40th birthday. Besides the trauma of this biological milestone (April 4th to be exact), I am also confronted with a disconcerting actuarial factoid. It seems my median remaining lifespan is also 40 years. In plain English, a healthy, non-smoking (OK, slightly overweight) 40-year-old male has a 50 percent chance of dying within the next 40 years, which is a 50 percent chance of surviving to the age of 80.

And, as is common around the halfway mark of the human lifecycle, in addition to thinking about aspirations for the future one tends to reflect on triumphs of the past. Besides my lovely wife and four daughters, one of the achievements I'm most proud of is a one-line formula that I developed and then published a few years ago with some colleagues of mine at York University. Yes, the hunting trophy on my mantle is a mathematical relationship with the pessimistic sounding name of the *probability of retirement ruin*.

First, some background. If we lived in a hypothetical world where everyone knew exactly how long they would live together with the rate of inflation during retirement as well as their precise investment portfolio return, then calculating sustainable spending rates would be easy. For example, a $1,000,000 nest egg at retirement can sustain a 3 percent inflation-adjusted spending rate of $50,000 per year for almost 30 years of retirement, as long as your portfolio earns a nominal 5.8 percent every year. If you want to spend $60,000 per year (i.e., a spending rate of 6 percent) then you must earn 7.3 percent per year. Stated differently, the present value of a $6 annual spending rate adjusted for 3 percent inflation over 30 years is $100, under a nominal 7.3 percent annual investment return.

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In practice, though, there are three major financial risks or threats that people face as they transition from wealth accumulation to retirement income and distribution. These three hazards are not accounted for in a deterministic framework, which obviously makes sustainability planning more complicated. The risks can be labeled as follows: (i) longevity risk; (ii) purchasing power risk; and (iii) investment risk. In other words, we don't know how long we will live; we don't know the precise rate of inflation; and our portfolio is destined to earn uncertain returns over time.

For a large number of retirees, these three risks are hedged or insured by defined benefit (DB) pension plans that provide lifetime income, insulated from the vagaries of the stock market. Many such plans include periodic adjustment to account for the increased cost of living. But for a substantial and growing fraction of soon-to-be retiring baby boomers, this classical protection is just not there. As everyone knows by now, DB pensions are being replaced with do-it-yourself savings, i.e. defined contribution 401(k) accounts and in some cases even the DB plans themselves are at the mercy of the bankruptcy process. Thus, many of us face the prospect of managing the three retirement risks ourselves — hopefully with the help of the financial services industry.

The mathematical formula I alluded to above attempts to relate or map these three risk factors into a summary risk metric called the probability of retirement ruin.

To use this formula you need to have a number of handy input factors. First, you need the estimated halfway mark for your retirement. This is the median remaining lifespan, or MRL. You can get this number from a longevity or mortality table. If you can't find it ask your doctor or insurance agent. As I mentioned above, my MRL is exactly 40 years. This doesn't necessarily imply that I am only planning to live for 40 more years, but rather that half my current cohort of 40-year-olds will reach the age of 80, while half will not.

Another important input factor is your inflation-adjusted retirement spending rate. This factor is usually denoted in percentage terms, for example 5 percent or 6 percent of your retirement nest egg. The third and final factor you need for computing the probability of ruin is the anticipated inflation-adjusted risk and return from your investment portfolio. I denote the expected return by the label AM and the volatility by the label VOL.

The following mathematical definition takes these four input factors — *MRL, AM, VOL, Spending* — and maps them into two summary variables.

$$\alpha = \frac{2xAM+2.773/MRL}{VOLxVOL+0.6931/MRL} - 1,$$

$$\beta = \frac{2xSpending}{VOLxVOL+0.6931/MRL}$$

Finally, use the retirement *alpha* and *beta* variables established by the above definition and look up the relevant number under the row and column from the following table to arrive at the retirement ruin probability.

I know this might appear confusing at first so here are some examples. Let's imagine that I decide to retire today at the age of 40, when my median remaining lifespan (MRL) is exactly 40 years. Remember that this implies a 50 percent chance I'll get to age 80. Now assume that I invest whatever savings or nest egg I have today into a portfolio that is expected to earn an arithmetic average (AM) of 8 percent after inflation in any given year and that the volatility (VOL) of my portfolio's return is 20 percent. These numbers — which are on the optimistic side — can usually be estimated from the historical performance of my investments and my current asset allocation. Finally, I desire an annual spending rate of 5 percent adjusted for inflation. Remember, this means that if I start with $1,000,000 then I'm spending $50,000 per year, and if I start with $100,000 then I'm spending $5,000 per year — both adjusted annually for inflation.

Bear with me here. According to the two mathematical definitions stated above, my retirement alpha is equal to

$$(0.16+0.0693)/(0.04+0.01732)-1=3.0 \text{ units}$$

and my retirement beta is equal to

$$(0.10)/(0.04+0.01732)=1.74 \text{ units.}$$

These are the intermediary ingredients for the main formula.

Finally, I take these two values and look up from the table the retirement ruin probability. In the alpha = 3 and beta = 1.74 case, the retirement ruin probability is approximately 26 percent. Thus, if I take early retirement today at the age of 40 my odds don't look very good. I will be spending too much, living too long, or simply not earning enough to achieve my sustainability goals. After all, a 26 percent ruin probability is only a 74 percent success rate.

In contrast, if I were to take early retirement and only spend 2 percent per year adjusted for inflation — which is $20,000 per $1,000,000 nest egg — the situation would look better. In this case my retirement alpha would be the same 3 units, but my retirement beta would be reduced to 0.70 units. My ruin probability would drop to less than 4 percent, which is over a 96 percent sustainability or success ratio. To me,

What is Your Retirement's Probability of Ruin?

Alpha \ Beta	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
4.00	0%	0%	1%	2%	4%	7%	10%	14%	19%	24%	30%
3.75	0%	0%	1%	3%	5%	9%	13%	18%	23%	29%	35%
3.50	0%	1%	2%	4%	7%	11%	16%	22%	28%	34%	40%
3.25	0%	1%	3%	6%	10%	15%	21%	27%	33%	40%	46%
3.00	0%	1%	4%	8%	13%	19%	26%	32%	39%	46%	52%
2.75	0%	2%	6%	11%	17%	24%	31%	38%	45%	52%	58%
2.50	1%	4%	9%	15%	22%	30%	38%	45%	52%	58%	64%
2.25	1%	6%	12%	20%	28%	37%	45%	52%	59%	65%	70%
2.00	3%	9%	17%	26%	36%	44%	52%	59%	66%	71%	76%
1.75	5%	14%	24%	34%	44%	52%	60%	67%	72%	77%	81%
1.50	8%	20%	32%	43%	52%	61%	68%	74%	79%	83%	86%
1.25	14%	28%	41%	53%	62%	69%	76%	81%	85%	88%	90%
1.00	22%	39%	53%	63%	71%	78%	83%	86%	89%	92%	94%
0.75	35%	53%	65%	74%	80%	85%	89%	91%	94%	95%	96%
0.50	52%	68%	78%	84%	89%	92%	94%	95%	97%	97%	98%

these risk metrics are acceptable, but then again living on $20,000 per year (i.e., a 2 percent spending rate) would be difficult.

Here is yet another example of the ruin probability for more realistic retirement ages and returns. Assume that you retire at the age of 65 with $1,000,000 and that your median remaining lifespan (MRL) is 25 years. Assume that you plan to withdraw $45,000 per year adjusted for inflation, which is a 4.5 percent spending rate. Also, assume your million-dollar nest egg is allocated to a mutual fund that is expected to earn 7 percent after inflation and all investment management fees. In this case the volatility or standard deviation of the portfolio will be taken as 15 percent. Now your retirement alpha is 4 units and your retirement beta is 1.79 units; both values are from the above definition. Finally, the relevant entry in the table indicates that your retirement ruin probability is approximately 10 percent. The sustainability is 90 percent.

Note that higher values of alpha are good for your retirement, as are lower values of beta. As you move to the lower right corner the ruin numbers increase and as you move to the upper left corner the ruin numbers decline. Intuitively this should make sense if you look carefully at the definitions of alpha and beta. Notice that higher spending rates increase your retirement beta (not good), while higher portfolio returns increase your retirement alpha (good). Also, greater investment volatility reduces your retirement alpha (not good), but also reduces your retirement beta.

Now that you have an intuitive feel for the table you might wonder where exactly the numbers came from. Or, how do you generate your own table with different values of alpha and beta? Actually, this is where the formula I mentioned earlier is used. The tabular values — which are not based on any Monte Carlo simulations — can be obtained in Microsoft Excel by typing the expression *GAMMADIST(beta,alpha,1, TRUE)*. Use the alpha and beta values I defined above, and out pops the precise number from the table.

If you want proof, check out volume 61(6) of the *Financial Analysts Journal* in which I elaborated on the calculus behind the formula, together with my colleague Chris Robinson. And, for those of you who remain skeptical that a simple analytic formula offers a shortcut around cumbersome and expensive Monte Carlo simulations of retirement income, I urge you to compare the properly calibrated *GAMMADIST* results against the output from your favorite financial planning software. Send your thanks to Bill Gates and his team at Excel.